Implementation of Biot's Model in OpenFoam

CE5890-Multiphysics

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Outline

- **Meaningful to know Poroealsticiy :**
  why I do
- **Governing equations of Poroelasticiy:**
  what I know
- **Implementation of Poroelasticiy in OpenFoam:**
  what I can do
- **Critical factors affecting the convergence:**
  what I learnt
- **Results validation:**
  how good I did
- **Conclusions:**
  what I got

Meaningful to know Poroelasticiy : why I do

Solid mechanics ⟷ Porous media ⟷ Pore fluids

Various applications

- Evaluate disposal of wastes in the subsurface: groundwater withdrawal
- Predict hazards related to the compaction of a producing oil reservoir, e.g., land subsidence and borehole damage
- Describe tumor-induced stresses in the brain
- Predict the bone deformation under a mechanical load

Governing equations of Poroelasticiy: what I know

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) = \alpha\nabla p - \mathbf{b}$$

Equilibrium

$$\phi\beta\frac{\partial p}{\partial t} + \alpha\frac{\partial}{\partial t}(\nabla\cdot\mathbf{u}) - \frac{k}{\eta}\nabla^2 p = f$$

Continuity
Darcy's law

where λ and μ are lames constants; α is the Biot coefficient; p is pore water pressure; $\mathbf{b}$ is body forces; $\mathbf{u}$ is medium displacements where ϕ is the porosity; β is the compressibility coefficient of fluid; k is the permeability of the porous medium; η is the dynamic viscosity of the fluid; and f is a scour term representing a forced extraction or injection process

$$\mathbf{u} \xleftrightarrow{\text{coupled}} p$$

Implementation of Poroelasticiy in OpenFoam

Develop the multi_solidDisplacementFoam solver

- Based on the existing solver: **solidDisplacementFoam**

- Set up the splits of implicit and explicit parts

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) = \alpha\nabla p - \mathbf{b} + \frac{\partial^2\mathbf{u}}{\partial t^2}$$

Implicit Explicit Explicit

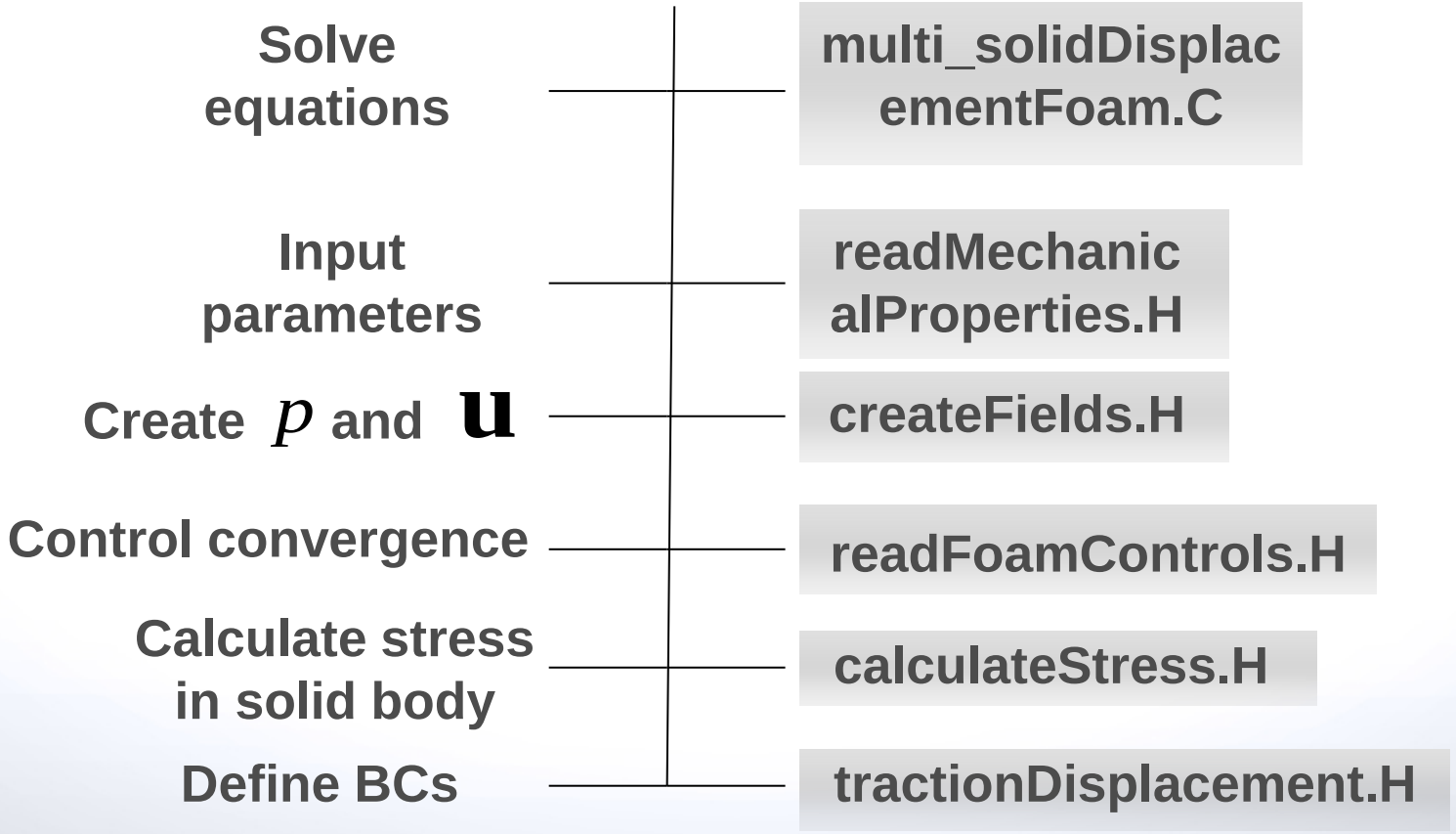
$$\phi\beta\frac{\partial p}{\partial t} + \alpha\frac{\partial}{\partial t}(\nabla\cdot\mathbf{u}) - \frac{k}{\eta}\nabla^2 p = f$$

Implicit Explicit Implicit

Implementation of Poroelasticiy in OpenFoam

Workflow of the multi_solidDisplacementFoam solver

multi_solidDisplacementFoam

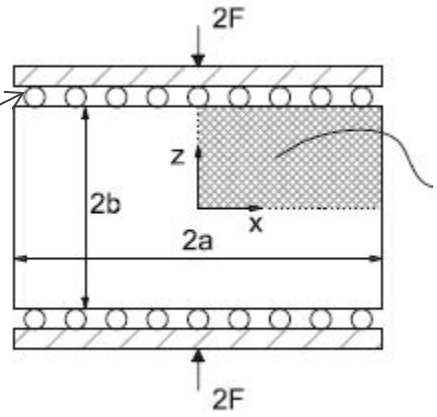


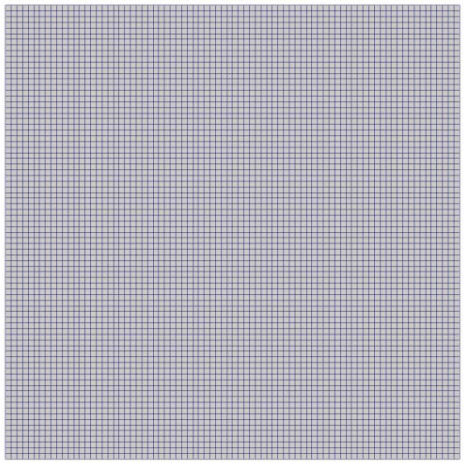
Implementation of Poroelasticiy in OpenFoam

Case study

- 2D poroelastic slab loaded by a constant vertical force (Ferronato et al., 2010)

Frictionless and impermeable





a=1 m
b=1 m
F=10000 N/m

Critical factors affecting the convergence

Case study

- Needed parameters

Parameters	values
k (m/s)	1e-5
ϕ	0.375
η (Pa s)	8.8e-4
μ (Pa)	40e6
λ (Pa)	40e6
α	1
B (Skempton's coefficient)	0.65
β (Pa^{-1})	?

$1/K$ ⟺ β

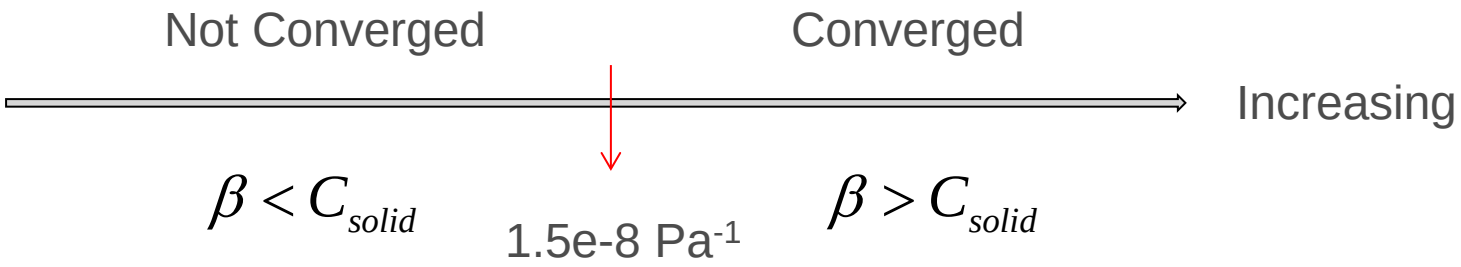
Solid compressibility (1e-8)

Fluid compressibility ?

Critical factors affecting the convergence

Case study

- Critical parameter: $\beta = 4.4e-10 Pa^{-1}$ in the reference



$$C_{solid} = \frac{1}{K} = \frac{2(1+\nu)\mu}{3(1-2\nu)}$$

Solid compressibility (1.5e-8 Pa^{-1})

$$\beta = 2.5e-8$$

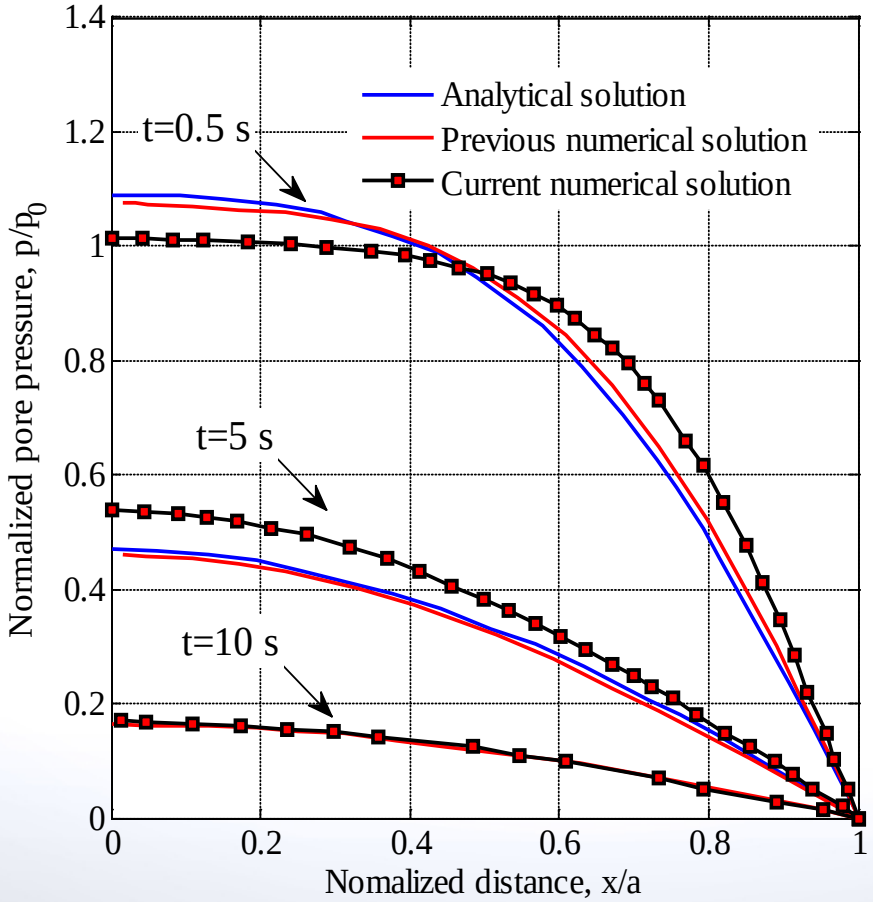
Fluid compressibility

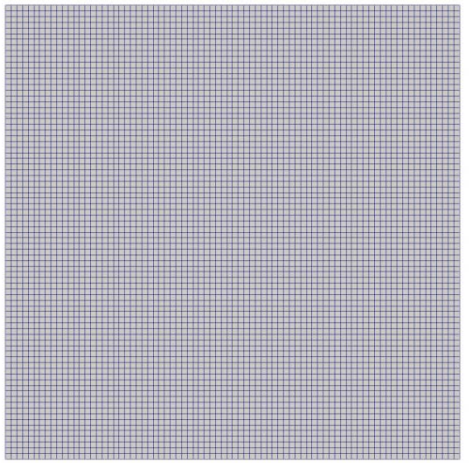
$$\phi\beta = 9.4e-9$$

Results validation

Case study

- Critical parameter: $\beta = 2.5e-8$





Pore pressure profile along the x-axis with time

Results validation

Case study

- Critical parameter: $\beta = 2.5e-8$

Variation of pore pressure (p) with respect to time within t=5 s

Results validation

Case study

- Critical parameter: $\beta = 2.5e-8$

Variation of displacement (u) with respect to time within t=5 s

Conclusions: what I learnt

- Key factor affecting the convergence in OpenFoam: the relative compressibility of the fluid to the solid

$\beta < C_{solid}$ Not Converged

$\beta \sim C_{solid}$ Not Converged or converged

$\beta > C_{solid}$ Converged

- Acceptable results but still remain unsolved issues

Questions?

Thank you