Formulation and Characterization of Freezing Saturated Soils

Zhen Liu, S.M.ASCE[1]; Xiong (Bill) Yu, M.ASCE[2]; Ye Sun[3]; and Bin Zhang[4]

Abstract: The writers have derived a unified governing equation for freezing saturated soils. This equation considers the governing mechanisms with respect to individual thermal and hydraulic fields. The writers included coupling effects such as the thermodynamic equilibrium on the water-ice interface. The morphology of the solid matrix and the physical chemistry of the water-ice interface have also been incorporated. The equation is comprised of terms with clear physical meanings. Typical properties that are indicative of freezing soils, e.g., segregation potential, can be derived from this equation. The writers discuss the material properties that are required for implementation of the equation. For the conventional parameters in the equation, i.e., thermal conductivity, heat capacity, and hydraulic conductivity, the corresponding mathematical descriptions were investigated. The functions for prediction of these parameters during the soil freezing process are presented. The writers have also proposed a relationship between the temperature and unfrozen water content for characterization of freezing saturated soils. The writers propose that the measurement of this relationship be conducted with a new technique that uses a thermo–time domain reflectometry (TDR) sensor. The writers have described detailed sensor and experiment designs for measuring this new relationship and have compared the results with data that were measured with a standard method.
DOI: 10.1061/(ASCE)CR.1943-5495.0000050.

[1]Graduate Assistant, Dept. of Civil Engineering, Case Western Reserve Univ., 2104 Adelbert Rd., Bingham 256, Cleveland, OH 44106-7201. E-mail: zhen.liu@case.edu

[2]Associate Professor, Dept. of Civil Engineering, Case Western Reserve Univ., 2104 Adelbert Rd., Bingham 206, Cleveland, OH 44106-7201 (corresponding author). E-mail: xxy21@case.edu

[3]Graduate Assistant, Dept. of Electrical Engineering and Computer Science, Case Western Reserve Univ., 2104 Adelbert Rd., Bingham 203C, Cleveland, OH 44106-7201. E-mail: yxs208@case.edu

[4]Civil Associate II, Michael Baker Inc., Centre City Tower, Suite 2010, 650 Smithfield St., Pittsburgh, PA 15222. E-mail: bxz26@case.edu

Note. This manuscript was submitted on December 8, 2011; approved on August 31, 2012; published online on September 3, 2012. Discussion period open until November 1, 2013; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Cold Regions Engineering***, Vol. 27, No. 2, June 1, 2013. © ASCE, ISSN 0887-381X/2013/2-94-107/$25.00.

CE Database subject headings: Frozen soils; Material properties; Saturated soils.

Author keywords: Freezing soil; Formulation; Unified governing equation; Characterization; Material property; Thermo–time domain reflectometry (TDR).

Introduction

Freezing saturated soils readily threaten the serviceability and sustainability of structures and facilities in permafrost and seasonally frozen regions. Freeze-thaw cycles that are associated with frost heaving or strength losses may cause extensive damage to various civil engineering structures, such as pavements and utility lines. For example, ice formation in a pavement foundation can lead to serious problems, e.g., an uneven uplift during freezing and then a loss of support when the ice lenses thaw (Konrad 1994).

Migration and crystallization of pore water in freezing soils are responsible for the soil behaviors. These effects not only alter the soil matrix directly, but also change other mechanical properties such as the elastic modulus. Various efforts have been made to understand the temperature-induced water flux in freezing soils. Attempts have also been made to develop characterization techniques for understanding the behaviors of freezing soils.

Philip and de Vries (1957) developed a thermodynamics-based theory to explain moisture movement in porous materials under temperature gradients. Most of the subsequent hydrodynamic models were developed based on this or similarly formulated theories (Guymon and Luthin 1974; Hansson et al. 2004; Harlan 1973; Noborio et al. 1996; Thomas et al. 2009). Cary (1965, 1966) summarized that surface tension, soil moisture suction, and kinetic energy changes that are associated with the hydrogen bond distribution, in addition to thermally induced osmotic gradients, should be responsible for thermally induced moisture flow. Based on this recognition, he made modifications to the theory of Philip and de Vries (1957). Dirksen and Miller (1966) used a similar concept but with an emphasis on mechanical analysis.

However, some researchers have formulated water and heat transport in response to a temperature gradient and a water pressure gradient, respectively, by using the theory of nonequilibrium thermodynamics (Groenevelt and Kay 1974; Kay and Groenevelt 1974; Taylor and Cary 1964). The theory of Kay and Groenevelt (1974) was developed by exploiting the energy dissipation equation and the Clapeyron equation for the three-phase relationship, and then obtaining transport equations from these equations. Theories that are based on nonequilibrium thermodynamics have rarely been employed in practice because of numerical implementation difficulties (Kay and Groenevelt 1974).

Miller (1978) proposed the rigid ice model. In his model, ice pressure was nonzero (in contrast with the assumptions of previous hydrodynamic models) and was related to water pressure through the Clapeyron equation. A variable of mean curvature was adopted, which was determined in accordance with the water content, hydraulic conductivity, and stress partition function. The movement of the freezing front as a function of mean curvature was manifested in the form of ice regelation

(Horiguchi and Miller 1980) with the liquid flux assumed. In summary, the rigid ice model assumes nonzero ice pressure and introduces the relationship between the mean curvature and other variables. These treatments, together with the Clapeyron equation and Darcy's law, laid the foundations for the multiphysics model.

Using a nonzero ice pressure, Gilpin (1980) developed a theory that is based on the assumption that water movement in the liquid layer is controlled by normal pressure-driven viscous flow. Water is drawn towards the base of the ice lens because of the curvature. This interfacial curvature, which inherently varies in porous materials, leads to the nonequilibrium conditions between the pressure and temperature in local freezing zones, such as the frozen fringe. Consequently, liquid water must move toward the ice lens to reach an equilibrium that is described by the Clapeyron equation.

Dash (1989) proposed an explanation that seems to be similar to that of Gilpin (1980) but actually differs. Dash (1989) attributed the driving force to the reduction of the interfacial free energy of a solid surface by a layer of the melted material, which occurs for all solid interfaces that are wetted by the melted liquid. Without a substrate, a mass flow occurred because of the difference in the thickness of the melted layer (liquid) along the interface between the liquid and solid layers. This resulted in a thermomolecular pressure to ensure equilibrium.

There are empirical models for engineering purposes, such as those of Konrad and Morgenstern (1980, 1981, 1982). In these models, the coupling effects were simplified by introducing an experimental relationship that the rate of water migration is proportional to the temperature gradient in the frozen fringe. The corresponding proportionality was termed segregation potential and was treated as an important property for characterizing freezing soils. The segregation potential depends on pressure, suction at the frost front, cooling rate, soil type, and other variables (Nixon 1991). Frost heave can be calculated once the segregation potential and other parameters, such as temperature gradients, are available. These models, which had been calibrated from experimental data, have enabled engineering frost heave calculations (Kujala 1997).

Several conclusions can be reached based on the previously noted efforts. First, coupling of the thermal field for heat transfer and the hydraulic field for water migration are responsible for the behavior of freezing soils from a physics viewpoint. These two coupled physical fields result in thermally induced water migration. The thermodynamic equilibrium between the water and ice is responsible for the coupling effects. However, material properties that are affiliated with either the thermal or hydraulic field change as freezing develops. The variations of these properties affect the coupling and ultimately change the coupled process. It is also possible that an intrinsic property of freezing soils, such as the segregation potential, governs the coupled thermohydraulic process.

This paper proposes a new theoretical formulation for freezing saturated soils. The writers present a unified governing equation for the coupled thermohydraulic process. The writers evaluated the material properties of freezing soils that are represented in the equation, and present the corresponding mathematical expressions for engineering applications. The writers propose a new relationship between the unfrozen water content and temperature for characterization of freezing saturated soils. The writers also describe a thermo–time domain reflectometry (TDR) sensor for measurement of this relationship.

Theoretical Formulation

Fourier's equation for heat conduction can be used for the heat transfer process in freezing soils. The convective term is not considered because of the comparatively smaller rate of water migration to that of heat conduction, and also because of the phase change of water. The energy that is absorbed or released by the phase change of water is treated as a heat sink or source on the right-hand side of the governing equation (Eq. 1). For simplicity, the writers adopted a one-dimensional form of Fourier's equation, as follows:

$$\rho C \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = \rho_w L \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \tag{1}$$

where ρ = soil density; ρ_w = density of water; C = gravimetric specific heat capacity of freezing soil; T = temperature (in kelvin); t = time; λ = thermal conductivity; L = latent heat of fusion of water; ρ_i = density of ice; and θ_i = volumetric ice content. x is the coordinate in the direction of gravity because freezing in soils typically occurs vertically.

The mixed-type Richards' equation is frequently used to describe fluid movement in variably unsaturated porous media. The equation exhibits good performance in ensuring mass conservation (Celia and Binning 1992). The writers added a term that is related to ice formation to the left-hand side to extend the Richards' equation to freezing saturated soils, as follows:

$$\frac{\partial \theta_w}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} + \frac{\partial}{\partial x}\left[\frac{K}{\rho_w g}\left(\frac{\partial \Psi}{\partial x} - 1\right)\right] = 0 \tag{2}$$

where θ_w = volumetric water content (or more specifically, unfrozen water content throughout this paper); K = hydraulic conductivity; g = gravitational acceleration; and Ψ = soil suction. The term inside the parentheses of Eq. (2) represents both the effect of soil suction and that of gravity on water migration.

The writers obtained the unified governing equation for the coupled thermohydraulic field in freezing saturated soils by substituting Eq. (1) into Eq. (2):

$$\frac{\partial \theta_w}{\partial t} + \frac{\rho C}{\rho_w L}\frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\frac{\lambda}{\rho_w L}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left[\frac{K}{\rho_w g}\left(\frac{\partial \Psi}{\partial x} - 1\right)\right] = 0 \tag{3}$$

There are three dependent variables, i.e., θ_w, T, and Ψ, in Eq. (3); thus, two additional relationships are required to guarantee the existence of a unique solution. One of these relationships is provided by the thermodynamic equilibrium between water and ice. This correlation has been repeatedly employed by previous models (Gilpin 1980; Groenevelt and Kay 1974; Kay and Groenevelt 1974; Philip and de Vries 1957). This equilibrium can be mathematically described with the generalized Clapeyron equation

$$\frac{d\Psi}{dT} = -\frac{\rho_w L}{T} \tag{4}$$

Eq. (4) can be integrated and modified to take the effects of solutes into consideration (Hansson 2005), and can be rewritten in the form of soil suction for freezing soils, as follows:

$$\Psi = -\rho_w L \ln \frac{T}{T_0} - \frac{\rho_w - \rho_i}{\rho_i} P_i - icRT \tag{5}$$

where T_0 = freezing point of bulk water (in kelvin) under a standard atmospheric pressure; P_i = ice pressure; i is the osmotic coefficient (or van't Hoff coefficient); c = solute concentration; and R is the universal constant.

Another relationship arises from the internal structures of soils and the physical chemistry of the interface between water and ice. There is a unique relationship between the water content and soil suction for any particular soil in the drying process (Fredlund and Xing 1994; van Genuchten 1980). Because of the similarity between the drying and freezing processes, a similar relationship can be described for the freezing process (Koopmans and Miller 1966; Spaans and Baker 1996), which expresses a unique relationship between the water content (θ_w) and soil suction. The slight difference between the two relationships arises from the fact that soil suction in drying and freezing processes corresponds with water-air and water-ice interfaces, respectively. Considering the relationships between the water content and soil suction, there should also be a unique correlation between the temperature and the water content, expressed in Eq. (6) as follows:

$$T = T(\theta_w) \tag{6}$$

If there is no ice lens, the water content is less than the porosity and the ice pressure can be assumed to be zero (Spaans and Baker 1996). By neglecting the effects of solutes and substituting Eq. (5) into Eq. (3), the writers obtained Eq. (7):

$$\frac{\partial \theta_w}{\partial t} - \frac{\partial}{\partial x}\left[\left(\frac{\lambda}{\rho_w L} + \frac{KL}{g}\frac{1}{T}\right)\frac{\partial T}{\partial x} + \frac{K}{\rho_w g}\right] = -\frac{\rho C}{\rho_w L}\frac{\partial T}{\partial t} \tag{7}$$

There are two components that contribute to the change of the liquid water content (accumulation term). One is a water flux term on the left-hand side of Eq. (7), and the other is a sink term on the right-hand side of Eq. (7), which results from the variation of temperature. Given that $\rho C/\rho_w L$ is generally much less than 0.01 according to documented data (Andersland and Ladanyi 2004), the importance of this term in Eq. (7) is far from significant. Therefore, the variation of the water content is primarily controlled by the water flux term that is itself caused by the temperature gradients, as expressed in Eq. (8) as follows:

$$J = -\left(\frac{\lambda}{\rho_w L} + \frac{KL}{g}\frac{1}{T}\right)\frac{\partial T}{\partial x} - \frac{K}{\rho_w g} \tag{8}$$

A physical representation of the segregation potential that was proposed by Konrad and Morgenstern (1980) can be obtained by integrating Eq. (8) over the frozen fringe. If the influence of gravity is neglected, the segregation potential is as follows:

$$\mathrm{SP} = \frac{l}{T_i - T_s}\frac{1}{l}\int_{x_s}^{x_s+l}(-J)dx = \frac{1}{T_i - T_s}\int_{T_s}^{T_i}\left(\frac{\lambda}{\rho_w L} + \frac{KL}{g}\frac{1}{T}\right)dT \tag{9}$$

where SP = segregation potential; T_s and T_i are the temperature on the top and base, respectively, of the frozen fringe [T_s was also termed segregation-freezing temperature by Konrad and Morgenstern (1980)]; x_s = ordinate of the top of frozen fringe; and l = length of the frozen fringe. The hydraulic conductivity is a function of the water content and thus a function of the temperature. As noted by Konrad and Morgenstern (1980), the segregation potential is dependent on the hydraulic conductivity and segregation-freezing temperature. This statement is well-supported by Eq. (9). However, if the average hydraulic conductivity of the frozen fringe ($\bar{K}$) is adopted, the segregation potential can be approximately reduced to Eq. (10):

$$\mathrm{SP} = \frac{\lambda}{\rho_w L} + \frac{\bar{K}L}{g}\frac{1}{T_s} \tag{10}$$

The experimental results of Konrad and Morgenstern (1981) fit the pattern of variation of SP with $\bar{K}$ that is described by Eq. (10).

The unified equation, Eq. (7), contains one dependent variable from the thermal field, T, and one dependent variable from the hydraulic field, θ_w. By employing the relationship that is expressed by Eq. (6), the unified equation can be further simplified to one dependent variable, as follows:

$$\frac{\partial T}{\partial t} - \frac{FL\rho_w}{L\rho_w + FC\rho}\frac{\partial}{\partial x}\left[\left(\frac{\lambda}{\rho_w L} + \frac{KL}{g}\frac{1}{T}\right)\frac{\partial T}{\partial x} + \frac{K}{\rho_w g}\right] = 0 \tag{11}$$

where F = derivative of T with respect to θ_w. F is also a function of water content as $F(\theta_w)$.

Eq. (11) can be solved once the boundary conditions are known and all of the material properties are identified. These material properties include the conventional parameters, i.e., thermal conductivity, specific heat capacity, and hydraulic conductivity, in addition to the proposed relationship between the temperature and water content. The latter is a new relationship for characterization of freezing soils, and is similar to a relationship that was derived for cement (Coussy and Monterio 2008) based on the Gibbs-Duhem equation. In the following section, the writers discuss methods for formulating the conventional parameters, and then describe a new technique for measurement of the temperature-water content relationship with a thermo-TDR sensor.

Characterization

The parameters in Eq. (11) describe the material properties of freezing soils for either the thermal or hydraulic field. They are influenced by the composition of the soils, i.e., the volume fractions of individual components, and thus these material properties keep changing during the freezing process. The material properties of individual components are constant. It is possible to predict the variations of these parameters with knowledge of only the water content (dependent) and the constant properties of soil components. In addition to these parameters, the writers have proposed a new relationship for characterization of freezing saturated soils in this paper. The existence of such a relationship has been proven, but there is no measurement method. The writers propose a new technique with a thermo-TDR sensor.

J. Cold Reg. Eng. 2013.27:94-107.

The next section presents the relevant theory, sensor design, materials, and measurement methods, in addition to the results, a discussion, and a validation.

Conventional Material Properties of Freezing Soil: Thermal Conductivity

Initial attempts at physically based models for the thermal conductivity of soils usually adopted a geometry in which inclusions in different shapes, e.g., cubic, spherical, ellipsoid, or lamellae, were well-arranged in a cubic lattice (de Vries 1963; Russell 1935; Woodside 1958). Among these models, the model proposed by de Vries (1963) was designated to unsaturated soils. It is now widely used, for example, in the simultaneous heat and water (SHAW) model (Flerchinger 2000). The model of de Vries (1963) was derived from the formulas for the electrical conductivity of a two-phase system that is comprised of uniform spheres of one material arranged in a cubic array of another material. According to Woodside (1958), de Vries (1963) adopted and extended the form by Burger (1915) and Eucken (1932) to the scenario of ellipsoidal particles and multiphase media. The equation was subsequently applied to partially frozen soils by Penner (1970) in the form of Eq. (12). This physically based model can be used to predict the thermal conductivity of freezing soils, as follows:

$$\lambda = \frac{\lambda_w \theta_w + \lambda_i \theta_i + \sum_{j=1}^{M} \lambda_j \theta_j F_j}{\theta_w + \theta_i + \sum \theta_j F_j} \tag{12}$$

where F_j = ratio of the average temperature gradient in the jth particles to the average temperature gradient in the continuous medium; and M = number of types of granules. Particles with the same shape and conductivity are considered as one type. The quantity F_j depends only on the shape and orientation of the granules, in addition to the ratio of the conductivity (λ_j/λ_w), as follows:

$$F_J = \frac{1}{3} \sum_{a,b,c} \left[1 + \left(\frac{\lambda_j}{\lambda_w} - 1 \right) g_a \right]^{-1} \tag{13}$$

where g_a (or g_b, g_c) = depolarization factor of the ellipsoid in the direction of a (or b, c) axis. The quantities g_a, g_b, and g_c depend on the ratios of the axes a, b, and c. Penner (1970) supported the use of $g_a = g_b = 0.125$ and $g_c = 0.75$, which was obtained by de Vries (1963) on a trial and error basis.

Conventional Material Properties of Freezing Soil: Heat Capacity

Heat capacity is usually formulated as the weighted sum of different components of a porous medium (Campbell 1985; de Vries 1963; Williams and Smith 1989). For freezing soils, the equation can be adjusted as follows:

$$C = \frac{1}{\rho} [\rho_w C_w \theta_w + \rho_i C_i \theta_i + \rho_s C_s (1 - \theta_w - \theta_i)] \tag{14}$$

where ρ = density of saturated soils; ρ_s = density of soil particles; and C_w, C_i, and C_s = specific heat capacity of water, ice, and soil particles, respectively.

Conventional Material Properties of Freezing Soil: Hydraulic Conductivity

Hydraulic conductivity is a function of permeability, water density, and viscosity, and is among the most challenging soil properties to model. Great attention has been paid to its prediction with theoretical approaches (Fredlund et al. 1994). Because of their similarity, the equations proposed for the hydraulic conductivity of unsaturated soils have also been frequently used for freezing soils (Hansson et al. 2004; Lundin 1990). Many researchers have tended to use an impedance factor in addition to the original equations to consider the effects of ice on the permeability (Hansson et al. 2004; Newman 1997). However, Newman (1997) and Watanabe and Wake (2008) have noted that the impedance factor was not necessary when an accurate soil water characteristic curve was available.

Fredlund et al. (1994) proposed that the hydraulic conductivity could be determined through the relative hydraulic conductivity, itself integrated from the soil water characteristic curve. For freezing soils, the relationship between water content and suction was used in place of the soil water characteristic curve in unsaturated soils. The water content-suction relationship can be easily developed based on the temperature-water content relationship, which is readily available with the proposed technique. The writers express the relative hydraulic conductivity as follows:

$$k_r(\Psi) = \frac{\int_{\ln(h)}^{b} \frac{\theta(e^y)-\theta(\Psi)}{e^y} \theta'(e^y)dy}{\int_{\ln(\Psi_{\mathrm{aev}})}^{b} \frac{\theta(e^y)-\theta_s}{e^y} \theta'(e^y)dy} \tag{15}$$

where $b = \ln(10,000,00)$ (Fredlund et al. 1994); y is a dummy variable of integration that represents the logarithm of suction; and Ψ_{aev} = air entry value of matric potential. The hydraulic conductivity at any water content can be obtained by multiplying the relative hydraulic conductivity with the hydraulic conductivity of the unfrozen saturated soil.

Characterization of Freezing Soil Using TDR: Theory

The writers used a thermo-TDR sensor for characterization of freezing saturated soils. The thermo-TDR is a fusion of a TDR module and a thermal measurement module. The TDR module provides accurate measurements of the water content in soils, whereas the thermal module with the thermocouples provides accurate measurements of temperature. The thermo-TDR sensor therefore offers a method to obtain water content and temperature data synchronously. The TDR sensor is capable of specifically measuring the quantity of unfrozen water in freezing soils, attributable to the significant drop of the dielectric constant of free water (approximately 81) as it changes into ice (approximately 3.2). Therefore, the writers used the degree of thawing (or freezing) as a real indicator of freezing status based on the physical nature of the freezing and thawing processes. The degree of thawing at any time t can be defined as the normalized unfrozen water content in accordance with Eq. (16), as follows:

$$\Gamma(\%) = \frac{w_t - w_f}{w_u - w_f} \times 100\% \tag{16}$$

where Γ = degree of thawing; w_u = gravimetric water content of a soil specimen at its completely unfrozen status; w_t = gravimetric water content of the freeze-thaw specimen at time t; and w_f = gravimetric water content in the completely frozen specimen. Ramo et al. (1994) and Liu et al. (2012) expressed Eq. (16) with TDR signals as follows:

$$\Gamma(\%) = \frac{\sqrt{K_{a,t}} - \sqrt{K_{a,f}}}{\sqrt{K_{a,u}} - \sqrt{K_{a,f}}} \times 100\% = \frac{L_{a,t} - L_{a,f}}{L_{a,u} - L_{a,f}} \times 100\% \tag{17}$$

where $K_{a,u}$ and $L_{a,u}$ = dielectric constant and apparent length, respectively, of the unfrozen specimen; $K_{a,t}$ and $L_{a,t}$ are the dielectric constant and apparent length, respectively, of the freezing specimen at time t; and $K_{a,f}$ and $L_{a,f}$ are the dielectric constant and apparent length, respectively, of the completely frozen specimen. The degree of freezing/thawing is also termed saturation in the results and discussion section. and will be compared with the saturation-suction relations that the writers measured in unsaturated soils.

Characterization of Freezing Soil Using TDR: Materials and Methods

The writers performed experiments on two representative types of subgrade soils in the state of Ohio (United States). Table 1 summarizes the index properties of these soils. The writers used a thermo-TDR sensor to measure the temperature and unfrozen water content simultaneously. Fig. 1 presents the geometry and components of the thermo-TDR sensor. The rods are 40 mm in length and spaced 6 mm apart. The diameter of the probe rod is 1 mm. Instead of solid rods as traditional TDR probes, the writers used hollow steel rods as the thermo-TDR probes. The writers installed one type-K thermocouple in each rod. The tubes were then backfilled with high thermal conductive epoxy.

The writers then designed experimental procedures for measurement of the temperature-water content relationship using a thawing process instead of a freezing process because of the ease of control of the thawing process in the writers' laboratory. Results for the thawing process can be assumed to be similar to that of the freezing process only if the hysteresis is negligible (Koopmans and Miller 1966; Spaans and Baker 1996).

The writers prepared a soil specimen with a high water content (nearly saturated) using a Harvard compactor, and installed the thermo-TDR probes into the soil specimens. The specimens and TDR were sealed with plastic wrap to prevent evaporation, and were then placed into a freezer at −18°C for approximately 24 h for complete freezing. The writers initiated thawing by cutting the power to the freezer with the TDR, and the temperature was monitored until the specimen completely thawed.

Table 1. Index Properties of the Soils Tested in This Study

Soil	Gravel	Coarse sand	Fine sand	Silt	Clay	Liquid limit	Plastic limit	Plastic index
1	7	5	10	28	50	25	14	11
2	10	7	10	14	59	40	18	22

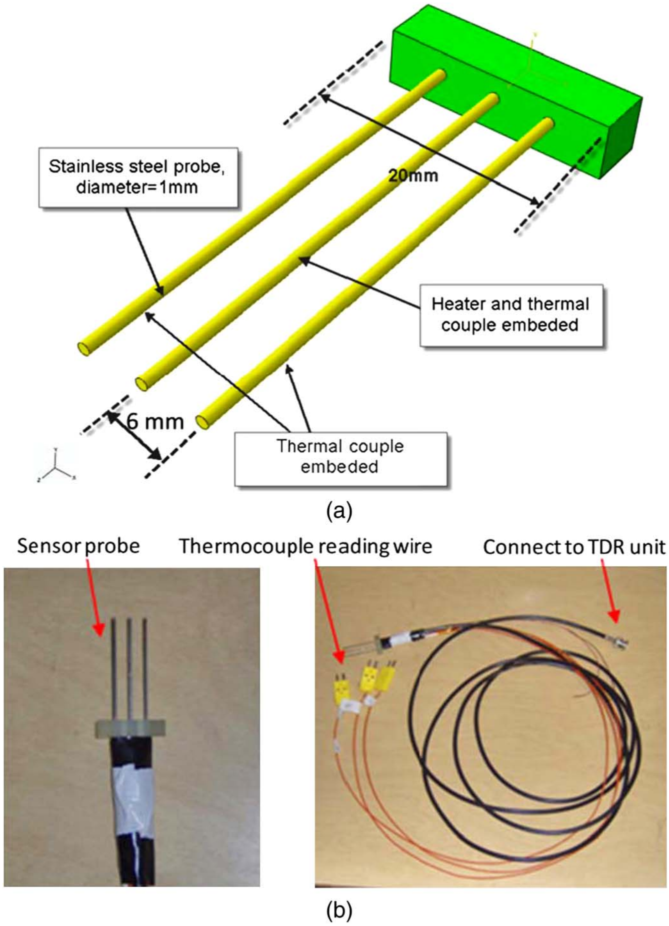

Fig. 1. Thermo-TDR probe: (a) schematic; (b) photographs

For soil number 1, the specimens were prepared using a Harvard Miniature compactor. The method of compaction, i.e., the mass of soil solids in each layer and extent of compaction energy, was carefully controlled to ensure that the specimen was uniform. The specimens were 71 and 35 mm in height and diameter, respectively. The writers mechanically inserted three probes to the full depth along the axis of the cylindrical soil specimens with the middle probe directly on the axis. Specimens that the writers prepared from soil number 2 were prepared in a different manner to determine if different geometries would affect the results. The writers used a steel ring with a height and inner diameter of 200 and 71 mm, respectively. The writers placed 100 g of soil in the ring on a steel table. The upper surface of the soil mass was flatted and then hammered for 12 blows using a 5-kg steel cylinder with a diameter of 71 mm, dropped from a height of 200 mm. The molded

J. Cold Reg. Eng. 2013.27:94-107.

soil specimens were cylinders with a diameter and approximate height of 71 and 12 mm, respectively. Because of the small heights of the soils specimens, the writers inserted the thermo-TDR into the specimen with the probes perpendicular to the axis of the cylinder, with the middle probe intersecting the axis. For each type of soil (i.e., soil numbers 1 and 2), the writers prepared duplicate specimens, among which one specimen was used for $T(\theta_w)$ measurements and the others were reserved for results validation using the filter paper method in accordance with ASTM D5298-03 (2003). For the specimens that were used for $T(\theta_w)$ measurements, the thawing processes were controlled such that they required more than 10 h. Accordingly, the rate of thermal exchange was less than 20 $\mathrm{W/m^2}$ during the thawing process, which was experimentally proven to be sufficient to ensure the quasi-state of thermodynamic equilibrium.

Characterization of Freezing Soil Using TDR: Results

Fig. 2(a) presents the measured temperature-saturation relationship ($T(S)$) for soil number 1. This relationship can be readily converted into the temperature-water content relationship, $T(\theta_w)$, which is required for solving the unified governing equation for freezing saturated soils [Eq. (11)]. The water content decreases as temperature decreases. The scattered points are distributed on an approximately smooth curve. To validate the measured results, the writers derived the saturation-suction relationship ($S(\Psi)$) from the measured $T(S)$ by employing the Clapeyron equation. The derived results of $S(\Psi)$ were then compared with those that were measured with the standard filter paper method in accordance with ASTM D5298-03 (2003), based on the similarity between the freezing and drying processes (Spaans and Baker 1996). Fig. 2(b) indicates that the saturation-suction relationship that the writers measured with the new technique and that of the standard filter paper method [ASTM D5298-03 (2003)] agree well. The writers performed another measurement on soil number 2 in accordance with the same procedures. Fig. 3(a) presents the measured $T(S)$ with the thermo-TDR sensor. The measured results were even better this time. The measured $T(S)$ is supported by the agreement

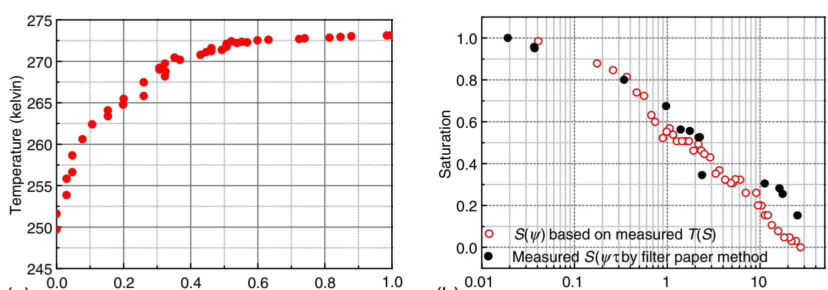

Fig. 2. Experiments on soil number 1: (a) measured temperature-saturation relationship, $T(S)$; (b) comparison of saturation-suction relationships from measured $T(S)$ and that by the standard filter paper method

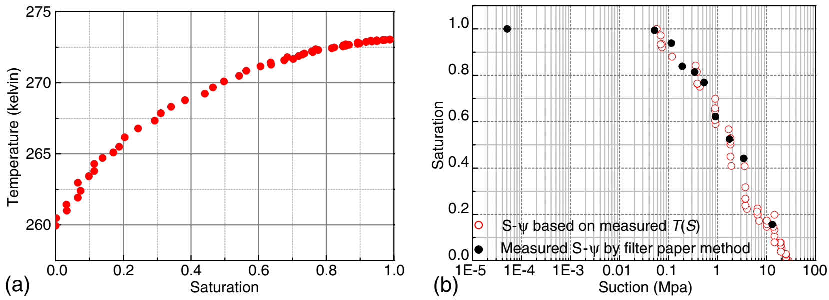

Fig. 3. Experiments on soil number 2: (a) measured temperature-saturation relationship, $T(S)$; (b) comparison of saturation-suction relationships from measured $T(S)$ and that by the standard filter paper method

between the derived $S(\Psi)$ from the measured $T(S)$ and that which the writers obtained with the standard filter paper method [ASTM D5298-03 (2003)].

Conclusion

Previous studies indicate that there is a unified equation for the coupled thermo-hydraulic field in freezing soils. The writers derived such a unified equation in this paper from the basic mechanisms and coupling effects. Typical properties for the characterization of freezing soils, such as segregation potential, can be derived from this equation. For implementation of the proposed equation, the writers discussed conventional parameters that represent material properties in thermal and hydraulic fields and presented their analytical formulations. Additionally, the writers proposed a new relationship for the characterization of freezing soils, and the writers also described and validated the measurement method. The writers expect that the behavior of freezing soils can be predicted by solving the unified governing equation with only the fundamental properties of soil constituents and the new relationship that the writers have proposed for freezing soil characterization.

References

Andersland, O. B., and Ladanyi, B. (2004). *Frozen ground engineering*, Wiley, Hoboken, NJ.

ASTM. (2003). “Standard test method for measurement of soil potential (suction) using filter paper.” *D5298-03*, West Conshohocken, PA.

Burger, H. C. (1915). “Das lertvermogen verdummter mischristallfreier lonsungen.” *Phys. Zs.*, 20(1), 73–76 (in German).

Campbell, G. S. (1985). *Soil physics with BASIC: Transport models for soil-plant systems*, 1st Ed., Elsevier, Amsterdam, The Netherlands.

Cary, J. W. (1965). “Water flux in moist soil: Thermal versus suction gradients.” *Soil Sci.*, 100(3), 168–175.

Cary, J. W. (1966). “Soil moisture transport due to thermal gradients: Practical aspects.” *Soil Sci. Soc. Am. Proc.*, 30(4), 428–433.

Celia, M. A., and Binning, P. (1992). "A mass conservative numerical solution for two-phase flow in porous media with application to unsaturated flow." *Water Resour. Res.*, 28(10), 2819–2828.

Coussy, O., and Monteiro, P. J. M. (2008). "Poroelastic model for concrete exposed to freezing temperatures." *Cem. Concr. Res.*, 38(1), 40–48.

Dash, J. G. (1989). "Thermomolecular pressure in surface melting: Motivation for frost heave." *Science*, 246(4837), 591–593.

de Vries, D. A. (1963). "Thermal properties of soils." *Physics of plant environment*, W. R. van Wijk, ed., North-Holland Publishing, Amsterdam, The Netherlands, 210–235.

Dirksen, C., and Miller, R. D. (1966). "Closed-system freezing of unsaturated soil." *Soil Sci. Soc. Am. Proc.*, 30(2), 168–173.

Eucken, A. (1932). "Heat transfer in ceramic refractory materials: Calculation from thermal conductivities of constituents." *Fortchg. Gebiete Ingenieurw. B*, 3(1), 353–360.

Flerchinger, G. N. (2000). "The simultaneous heat and water (SHAW) model: Technical documentation." *Technical Rep. NWRC 2000-09*, USDA Agricultural Research Service, Boise, ID.

Fredlund, D. G., and Xing, A. (1994). "Equations for the soil-water characteristic curve." *Can. Geotech. J.*, 31(4), 521–532.

Fredlund, D. G., Xing, A., and Huang, S. (1994). "Predicting the permeability function for unsaturated soils using the soil-water characteristic curve." *Can. Geotech. J.*, 31(4), 533–546.

Gilpin, R. R. (1980). "A model for the prediction of ice lensing and frost heave in soils." *Water Resour. Res.*, 16(5), 918–930.

Groenevelt, P. H., and Kay, B. D. (1974). "On the interaction of water and heat transport in frozen and unfrozen soils: II. The liquid phase." *Soil Sci. Soc. Am. Proc.*, 38(3), 400–404.

Guymon, G. L., and Luthin, J. N. (1974). "A coupled heat and moisture transport model for arctic soils." *Water Resour. Res.*, 10(5), 995–1001.

Hansson, K. (2005). "Water and heat transport in road structures—Development of mechanical models." Ph.D. thesis, Uppsala Univ., Uppsala, Sweden.

Hansson, K., Simunek, J., Mizoguchi, M., Lundin, L. C., and van Genuchten, M. T. (2004). "Water flow and heat transport in frozen soil: Numerical solution and freeze-thaw applications." *Vadose Zone J.*, 3(2), 693–704.

Harlan, R. L. (1973). "Analysis of coupled heat-fluid transport in partially frozen soil." *Water Resour. Res.*, 9(5), 1314–1323.

Horiguchi, K., and Miller, R. D. (1980). "Experimental studies with frozen soil in an 'ice sandwich' permeameter." *Cold Reg. Sci. Technol.*, 3(2), 177–183.

Kay, B. D., and Groenevelt, P. H. (1974). "On the interaction of water and heat transport in frozen and unfrozen soils: I. Basic theory: The vapor phase." *Soil Sci. Soc. Am. Proc.*, 38(3), 395–400.

Konrad, J. M. (1994). "Sixteenth Canadian geotechnical colloquium: Frost heave in soils: Concepts and engineering." *Can. Geotech. J.*, 31(2), 223–245.

Konrad, J. M., and Morgenstern, N. R. (1980). "A mechanistic theory of ice lens formation in fine-grained soils." *Can. Geotech. J.*, 17(4), 473–486.

Konrad, J. M., and Morgenstern, N. R. (1982). "Prediction of frost heave in the laboratory during transient freezing." *Can. Geotech. J.* 19(3), 250–259.

Konrad, J. M., and Morgenstern, N. R. (1981). "The segregation potential of a freezing soil." *Can. Geotech. J.*, 18(4), 482–491.

Koopmans, R. W. R., and Miller, R. D. (1966). "Soil freezing and soil water characteristic curves." *Soil Sci. Soc. Am. Proc.*, 30(6), 680–685.

Kujala, K. (1997). "Estimation of frost heave and thaw weakening by statistical analyses." *Proc., 7th Int. Symp. on Ground Freezing and Frost Action in Soils*, S. Knutsson, ed., Balkema, Rotterdam, The Netherlands, 31–41.

Liu, Z., Yu, X. B., Yu, X., and Gonzalez, J. (2012). "Time domain reflectometry sensor-assisted freeze/thaw analysis on geomaterials." *Cold Reg. Sci. Technol.*, 71(1), 84–89.
Lundin, L. C. (1990). "Hydraulic properties in an operational model of frozen soil." *J. Hydrol.*, 118(1), 289–310.
Miller, R. D. (1978). "Frost heaving in non-colloidal soils." *Proc., 3rd Int. Conf. on Permafrost*, National Research Council of Canada, Ottawa, 708–713.
Newman, G. P., and Wilson, G. W. (1997). "Heat and mass transfer in unsaturated soils during freezing." *Can. Geotech. J.*, 34(1), 63–70.
Nixon, J. F. (1991). "Discrete ice lens theory for frost heave in soils." *Can. Geotech. J.*, 28(6), 843–859.
Noborio, K., McInnes, K. J., and Heilman, J. L. (1996). "Two-dimensional model for water, heat and solute transport in furrow-irrigated soil: I. Theory." *Soil Sci. Soc. Am. J.*, 60(4), 1001–1009.
Penner, E. (1970). "Thermal conductivity of frozen soils." *Can. J. Earth Sci.*, 7(3), 982–987.
Philip, J. R., and de Vries, D. A. (1957). "Moisture movement in porous materials under temperature gradients." *Trans. Am. Geophys. Union*, 38(2), 222–232.
Ramo, S., Winnery, J. R., and van Duzer, T. (1994). *Fields and waves in communications electronics*, 3rd Ed., Wiley, Hoboken, NJ.
Russell, H. W. (1935). "Principles of heat flow in porous insulators." *J. Am. Ceram. Soc.*, 18(1–12), 1–5.
Spaans, E. J. A., and Baker, J. M. (1996). "The soil freezing characteristic: Its measurement and similarity to the soil moisture characteristic." *Soil Sci. Soc. Am. J.*, 60(1), 13–19.
Taylor, S. A., and Cary, J. W. (1964). "Linear equations for the simultaneous flow of matter and energy in a continuous soil system." *Soil Sci. Soc. Am. Proc.*, 28(2), 167–172.
Thomas, H. R., Cleall, P., Li, Y. C., Harris, C., and Kern-Luetschg, M. (2009). "Modelling of cryogenic processes in permafrost and seasonally frozen soils." *Geotechnique*, 59(3), 173–184.
van Genuchten, M. T. (1980). "A close-form equation for predicting the hydraulic conductivity of unsaturated soils." *Soil Sci. Soc. Am. J.*, 44(5), 892–898.
Watanabe, K., and Wake, T. (2008). "Capillary bundle model of hydraulic conductivity for frozen soil." *Water Resour. Res.*, 44(12), 1927–1932.
Williams, P. J., and Smith, M. W. (1989). *The frozen earth: Fundamentals of geocryology*, Cambridge University Press, Cambridge, UK.
Woodside, W. (1958). "Calculation of the thermal conductivity of porous media." *Can. J. Phys.*, 36(7), 815–822.

J. Cold Reg. Eng. 2013.27:94-107.